FSO-based Small Satellite Systems with Quad-Cell: Channel Modelling and Performance Evaluation

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Abstract

In this paper, we present a channel model for ground-to-CubeSat free space optical links that utilizes a quad-cell receiver. This model measures the channel gain between the lens and each of the photodiodes on the quad-cell. We assess link performance to determine the optimal receiver size for minimizing outage probability. Our findings facilitate the development of accurate spatial beam tracking and improved data detection, promising advancements in CubeSat network communication.

Introduction

Satellite communication using traditional commercial bandwidth allocations (X, Ku, and Ka bands) faces challenges meeting the demands of data-intensive 6G and beyond. To address this, satellite operators are turning to optical communication to alleviate the impact of limited radio frequency spectrum [1]. More recently, CubeSats have emerged as a cost-effective solution [2]. These platforms aim to enhance data transmission rates and reduce payload size. As a result, they make it feasible to use off-the-shelf components in ground-based optical networks, ultimately reducing costs [3]. An effective and commercially available component for achieving these objectives is the quad array of photodetectors, also known as the quad cell, which can be used at the receiver [4-5]. This component, which usually consists of avalanche photodiodes for long-range satellite links, enables simultaneous optical beam position sensing across a broader field of view while also providing multiplexing gain.

To assess the benefits of these systems, one should clearly understand the communication channel. There are challenges like angle of arrival fluctuations due to CubeSat vibrations resulting from imperfectly stabilized commercial products [2]. Our primary goal is to introduce a comprehensive channel model for ground-to-CubeSat free space optical (FSO) links using a quad-cell receiver. This model extends beyond traditional FSO models by incorporating additional parameters, specifically the channel coefficient between the receiver lens and the detectors on the quad-cell. In our proposed model, compared to the previous related works, we can calculate precise power distribution on individual detectors in the array. This greatly improves data detection accuracy and spatial beam tracking. We use this model to assess link performance, with a particular focus on outage probability and the influence of quad-cell size. Our proposed model serves as a foundation for future research in this field.

System Model for Quad Detector Satellite Link

FSO channel model (from an optical source to the receiver lens) has been extensively characterized within the literature under various circumstances [6], as delineated below

$$h_{\mathrm{c}} = h_{\mathrm{al}} h_{\mathrm{at}} h_{\mathrm{pl}}, \tag{1}$$

Here, h_{al} signifies the atmospheric attenuation, h_{at} denotes the atmospheric turbulence, h_{pl} accounts for the geometrical loss attributed to pointing errors. The attenuation of light as it propagates through the atmosphere is a consequence of both absorption and scattering phenomena, and this attenuation for a link with length L is quantified by the exponential expression known as the Beer-Lambert Law as

$$h_{al} = e^{-\alpha L} \tag{2}$$

where $\alpha = \left(\frac{3.91}{V}\right)\left(\frac{\lambda(\mathrm{nm})}{\lambda_0}\right)^{-q}$. Here, λ is the optical wavelength, $\lambda_0 = 550$ nm and V is the visibility in kilometers. According to Kim's model [7], for a satellite-based link exceeding 50 km in length, the value of q is the size distribution of the scattering particles, and it is set to 1.6.

The Gamma-Gamma (GG) distribution serves as a suitable choice for accurately modeling atmospheric turbulence as outlined below

$$f_G(h_{at}) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_{at}^{\frac{\alpha+\beta}{2}-1} k_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_{at}}\right) \tag{3}$$

where, α and β respectively refer to the effective number of large-scale and small-scale eddies and they are determined based on Rytov variance. Additionally, when the received beam waist significantly exceeds the receiver aperture, i.e., $\omega_z \gg r_a$, as is the case in our ground-to-satellite link scenario, the optical beam at the receiver predominantly maintains its plane wave characteristics [8]. Consequently, the pointing loss can be simplified to:

$$h_{pl} = A_0 \exp\left(\frac{-2r_d^2}{\omega_{z_{\mathrm{eq}}}}\right) \tag{4}$$

where r_d is the radial beam displacement, which signifies the separation distance between the center of the optical beam footprint and the center of the receiver aperture, caused by pointing errors. We also have $\omega_{z_{\mathrm{eq}}}^2 = \omega_z^2 \frac{\pi \mathrm{erf}(\vartheta)}{2\vartheta \exp(-\vartheta^2)}$, $A_0 = [\mathrm{erf}(\vartheta)]^2$, and $\vartheta = \left(\sqrt{\pi} r_a\right)\left(\sqrt{2}\omega_z\right)$. For a satellite-based links pointing errors arise from various factors, such as beam wandering and receiver position vibrations.

When transmitting an optical signal from a ground node to the satellite, the aperture lens captures the received signal. Subsequently, the collected signal passes through the lens and is focused onto the quad-cell within the lens aperture area. However, CubeSat orientation fluctuations cause the Rx lens's center to deviate from the center of the received optical beam, resulting in angle-of-arrival (AoA) fluctuations, as depicted in Fig. 1. These AoA fluctuations create a shifted diffraction pattern (image beam dancing) at the photodetector array, which can attenuate the received optical power. Therefore, it is essential to consider an additional channel parameter, h_{lq}, to address the channel coefficient between the receiver lens and the quad-cell.

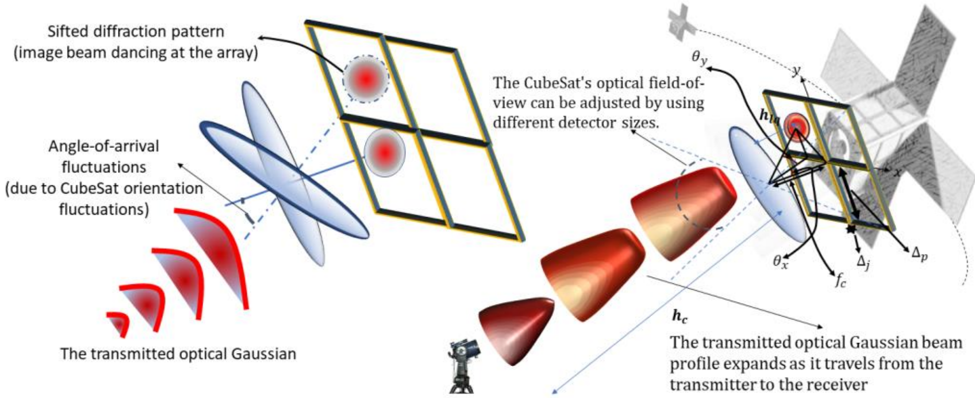

Fig. 1. Left: Deviated received beam on the quad-cell, right: definition of parameters

We assume that the Rx lens and the quad-cell are positioned within the x-y plane, while the beam propagates along the z-axis. To calculate h_{lq}, one can derive the intensity of the incident optical beam on the quad-cell as a two-dimensional Gaussian-shaped function, which is expressed as follows

$$I_p(x,y) = \frac{h_1 P_t}{2\pi\sigma_I^2} \exp\left(-\frac{(x-f_c\theta_x)^2+(y-f_c\theta_y)^2}{2\sigma_I^2}\right). \tag{5}$$

Here, θ_x and θ_y represent the random deviations of the received laser beam caused by the fluctuations of the receiver in the x-z and y-z planes respectively, with variance equal to σ_f^2. Also, σ_I^2 is the variance of the intensity of the optical beam. Let denote Δ_j as the junction width (dead space) and Δ_p as the active width of each photodetector in the quad cell. Consequently, the channel coefficient from the aperture lens to the corresponding (i,j)-th detector can be derived as follows

$$h_{lq,ij} = \int_{(i-2)\Delta_p+\Delta_j/2}^{(i-1)\Delta_p-\Delta_j/2} \int_{(j-2)\Delta_p+\Delta_j/2}^{(j-1)\Delta_p-\Delta_j/2} \frac{1}{2\pi\sigma_I^2} \times \exp\left(-\frac{(x-f_c\theta_x)^2+(y-f_c\theta_y)^2}{2\sigma_I^2}\right) \mathrm{d}x\mathrm{d}y \tag{6}$$

$$= \left[Q\left(\frac{(i-2)\Delta_p+\Delta_j/2-f_c\theta_x}{\sigma_I}\right) - Q\left(\frac{(i-1)\Delta_p-\Delta_j/2-f_c\theta_x}{\sigma_I}\right)\right] \times \left[Q\left(\frac{(j-2)\Delta_p+\Delta_j/2-f_c\theta_y}{\sigma_I}\right) - Q\left(\frac{(j-1)\Delta_p-\Delta_j/2-f_c\theta_x}{\sigma_I}\right)\right].$$

Finally, to obtain the end-to-end channel coefficient associated with the (i, j)-th detector we have

$$h_{\mathrm{ij}} = h_{\mathrm{c}} h_{\mathrm{lq,ij}}. \tag{7}$$

Fig. 2 shows the PDF of equal gain combining (EGC) scheme over the channel under different fluctuation variances. As receiver fluctuations increase, the PDF curves skew towards lower h_{EGC} values, indicating a higher probability of outcomes in that range. In other words, when receiver fluctuations become more pronounced, it is an indication that the channel conditions are deteriorating. This deterioration leads to poorer performance, as lower values of h_{EGC} are associated with worse channel conditions.

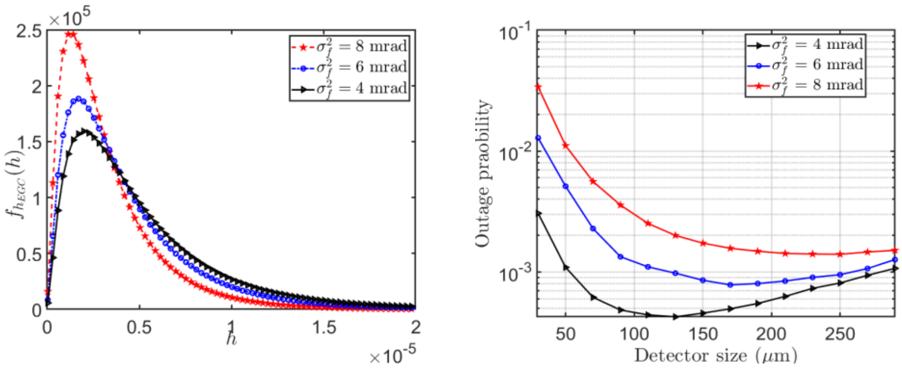

Fig. 2. Left: The PDF of EGC channels under different fluctuation variances, right: Outage probability vs. the detector size, Δ_p, under different fluctuation variances

Numerical Results

In this section, following a thorough channel modeling, we assess link performance using the outage probability metric, defined as follows:

$$\mathbb{P}_{\text{out}} \quad = \text{Prob}\{\gamma(h) < \gamma_{\text{th}}\} \tag{8}$$

where γ_{th} represents the SNR threshold. Here, the SNR is defined as

$$\gamma(h) = \frac{R^2 P_t^2 h^2}{\sigma_n^2} \tag{9}$$

where R represents the photodetector responsivity, P_t is the transmit power, $h = \ h_{\text{EGC}}$ is the instantaneous EGC channel gain, and σ_n^2 is the noise varince, which includes both thermal and background noises. Fig. 2 shows the outage probabilty curves versus detector sizes under diffent values of σ_f^2. As seen in the figure, increasing detector size improves link reliability, particularly by widening the receiver's field-of-view to mitigate the adverse impact of CubeSat random fluctuations. However, beyond an optimal point, further enlarging the detector size does not necessarily enhance link reliability. In particular, enlarging the receiver field-of-view by increasing the detector size results in accepting more desired transmit power as well as undesired background noise. Consequently, beyond an optimal point for the detector size (i.e., an optimal Rx field-of-view), the amount of background noise becomes dominant over the signal level, leading to a reduced SNR and an increased outage probability.

Conclusions and Future Research Direction

This research has introduced a novel channel parameter that allows for the precise distribution of power on individual detectors within the array. This not only enhances our understanding of the communication channel but also opens opportunities to several promising research directions and practical applications, including:

- Improved Data Detection: The ability to accurately distribute power on each detector empowers more robust and accurate data detection techniques, ensuring higher data reception accuracy in CubeSat-based networks.

- Accurate Spatial Beam Tracking: Leveraging this insight into power distribution, we can develop sophisticated spatial beam tracking methods, allowing for precise tracking in dynamic space environments.

- Array Extensibility: While our study focuses on quad arrays, this concept can seamlessly extend to arrays of arbitrary sizes, promising wider coverage and improved sensitivity.

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